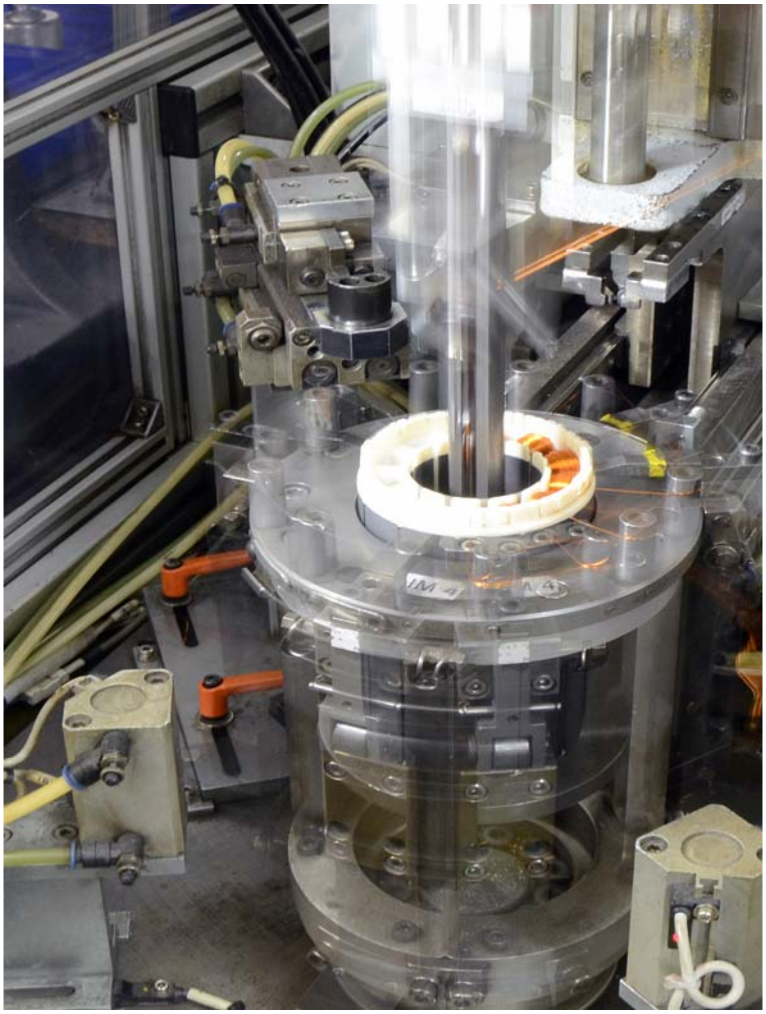

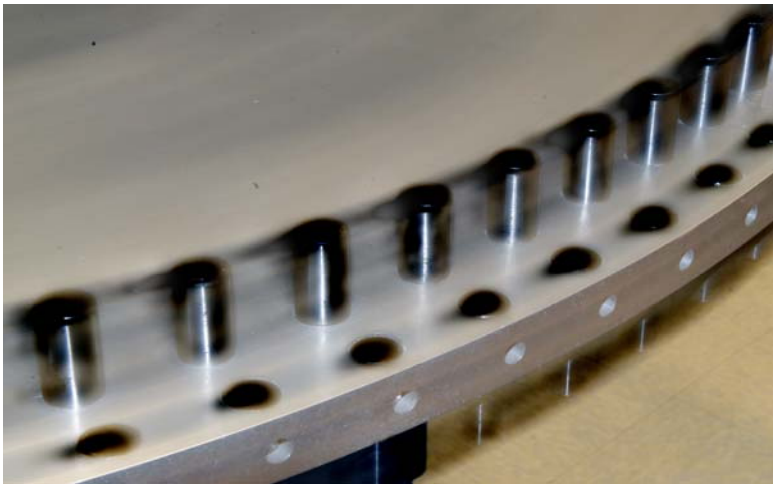

Energy Management of a Servomotor: Effects of Inertia Ratio

KOLLMORGEN

Because Motion Matters™

White Paper

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Your first question may be what is a servomotor's inertia ratio or mismatch (*J_load : Jm* or *J_load/Jm*)? Simply put, the inertia ratio is an important figure-of-merit for helping the machine designer consider servo controllability and risk of future potential control instabilities during the design's initial inception. While an important figure-of-merit for all closed-loop (servo) applications, it is perhaps even more relevant for the most dynamic servo applications.

This article presents an in-depth look at how the servomotor inertia ratio or mismatch evolved with the onset of servo drive technology and its significant impact on energy savings by addressing:

1. Why the newest digital servo drive and feedback technologies can be used to achieve higher inertia ratios (J_*load : Jm*) while maintaining stable control to a targeted velocity/position?
2. How the newest drive and feedback capabilities can enable machine design for the most efficient power utilization?
3. Why most dynamic indexing type applications can achieve these potential energy savings?
4. Why proper motor-drive-feedback selection of a servo controlled axis is possibly the single most significant savings element a machine designer can make for reducing the user's operational cost?

The moment of inertia ratio or mismatch for a rotary servo system can be divided into two parts defined as (1) the total moment of inertia of the load (*J_load*) of all the axis' components (reflected back through the mechanism when applicable) at the motor's shaft summed together, and (2) the motor's moment of inertia (*Jm*). An inertia mismatch however is not a concrete number or even a concrete range for every application. Although with some experience around the specific design and utilization of a given technology, one ratio range may be found to be more applicable to specific applications than other ratios.

Many articles and technical manuals present the ideal inertia mismatch to be 1:1. But this is the ideal mismatch for maximum power transfer, while the acceleration and deceleration energy is evenly split between J_load and Jm (where *J_load = Jm* and *J_total = 2*J_load*). Bearing in mind other design issues, this 1:1 mismatch minimizes potential control issues while maximizing energy utilization for dynamic application.

To obtain the most efficient power utilization, the machine designer should look for maximum acceleration of the Load inertia, while considering limitations and capabilities for the axis' stability, controllability, performance, accuracy, repeatability and so on. The maximum acceleration for a fixed J_load can best be achieved with a minimal *Jm* (not a matched *load: Jm*), resulting in the lowest possible power requirements.

History of Servomotor Inertia Ratio or Mismatch as Important Factor-of-Merit

Early servo drives based on analog technology had limited capabilities. This meant the standard compensation (COMP) or marriage between the drive, motor and the anticipated mechanism (servo control loop tuning) was done by hand adjustments, manually with resistance and capacitance decade boxes in a lab environment that involved an oscilloscope. As a result, these efforts made it difficult to specifically adjust or fine tune the servo's control loops against any customer specific mechanism.

The drive manufacturer needed to purpose with each motor-drive combination, a predetermined standard COMP that would present the best possible axis stability for each specific motor-drive combination, where the standard COMP would fulfill the majority of application requirements (Risk Management). The best way to accomplish this was to safely anticipate the customer's mechanism with an ideal inertia mismatch (*J_load : Jm*) of 1:1. This ratio presented the least risk of control loop stability issues by the utilization of maximum power transfer equations.

To imagine one of the major stability issues, think of a mechanically advantaged axis with some minimal backlash within a gearhead mounted at the servomotor. A standard COMP needed to be capable of maintaining current loop, velocity loop and position loop stability through the reflected inertia range that the motor would see. This reflected inertia range meant the motor will not only see the total maximum reflected inertia of the load, but also the minimum value seen during the normal transition of a drive tooth, between driven teeth within the gearhead. The closer the axis' machine design was to the anticipated (presumed) inertia mismatch 1:1, the more likely axis stability would be maintained during operation.

So drive manufacturers reasonably chose a standard compensation inertia mismatch and directed customer designs to have a specific range of inertia mismatch (factor-of-merit) in the marketplace in such a way that their standard COMPs would be stable for most applications. The ideal 1:1 (*J_load : Jm*) mismatch based on the maximum power transfer equations provided a level of assurance that a specific motor-drive combination would work in an application with only gain adjustments of the current and/or velocity loop (and position-loop when applicable).

Typically these analog current and velocity loop compensations with a 1:1 (*J_load : Jm*) mismatch would be good with little risk of control issues, up to about 3-5:1. After this inertia mismatch of about 3-5:1, the control-loops become more application dependent up to about 3-8:1 (gray area range), or even upwards of 10:1 in some cases. After this gray and application dependent range, one was almost assured regardless of the application type that a special compensation would be required. Gray area and higher inertia mismatch COMPs (specials) were done to accommodate specific application requirements.

Having a defined standard inertia mismatch, with a desirable inertia range (factor-of-merit) for the drive manufacturer and machine designer, allowed servo systems to be sold with little risk of the COMP not meeting most application requirements. In turn, this kept everyone (customer, supplier and manufacturer) from going crazy over stability and control issues! Many, if not most, analog drive manufacturers utilized this 1:1 inertia (maximum power transfer) ratio for their standard COMPs, though their suggested inertia mismatch range of *J_load : Jm* (factor-of-merit) may have varied as a function of their experience, market, and the drive's control-loop transfer function. Assuming a good mechanical servo designed axis, this standard 1:1 inertia COMP by the first (analog) servo drive manufacturers typically presented a stable (controllable) axis for most applications. At that time, inertia ratios upward of 3-5:1 were common and a ratio range at 1:1<= (*J_load : Jm)* <= 3:1 was typical for many dynamic high speed indexing applications.

Based on this information, the inertia mismatch 1:1 was widely used as the resulting solution to meet the marketing needs of the drive manufacturers. This allowed a complicated product to be sold into the market place with minimal risk of control instabilities, thus maximizing customer satisfaction by minimizing potential issues. Even most stepper motor drives with motor specifications of that time were promoted by their manufacturers in a similar manner utilizing a specific inertia ratio to present motor-drive capabilities; and everything worked fine for this open-loop system as long as the actual application load inertia and frictions were close to or less than those utilized to determine the open loop system capability and data publication specifications.

Even though most servo motion control axes that approach a 1:1 inertia mismatch are less likely to have control-loop instabilities, an application's ideal moment of inertia ratio is much more fluid than a fixed number or range. That is, each axis' ideal inertia mismatch depends on its mechatronic solution, application and components which include but are not limited to: the specific axis motion profile and dynamics, friction, stiction, external loading, backlash, compliance and stiffness, loads, mechanism inertia, feedback resolution, number of moving bodies between the load and motor, natural frequencies of the design, and the motor's drive PWM/SVM and update rates (and separate controller update rates, when applicable). These design factors are not typically or fully discussed within manuals and articles during inertia mismatch (*J_load : Jm*) considerations simply due to their complexity and history of how things have come about! However they all come into play for the ultimate operation and stability of an axis' closed-loop mechanism.

New Capabilities

When digital drives for servomotors were first introduced to the market place, it was a giant step for compensation flexibility, filtering and programming motion profiles, but little changed with respect to those other items affecting the figure-of-merit (inertia mismatch). Additionally, many of the analog controlled high performance applications could not be replaced with the digital servo drives of that time for multiple reasons. Today, the newest digital servo drive technologies have a rich set of software and hardware features and capabilities for an enhanced human interface experience and offer remarkable compensation flexibility. They feature higher update rates and higher resolution feedback devices in excess of 2^21 bits per revolution upwards of 2^27 bits, resulting in a more responsive servo bandwidth (BW) for most applications. For example, a servo axis that worked in the past, generating a feedback resolution of 2^12 or 2^16 counts per mechanical revolution, can now have the same number of counts in a fraction of the previous time or displacement. This new capability allows for higher control-loop gains which lead to higher BWs for catching and controlling possible instabilities before they become unstable.

These faster processor speeds, faster update rates, and higher feedback resolutions with the best mechatronic designs present noticeably higher control capability and allowing us in the servo industry, perhaps for the first time, to have enough control over a typical dynamic application to assume the effects of the figure-of merit (*J_load: Jm*) to be minimal. Thus enabling the industry to pinpoint an inertia ratio range for an axis' most efficient energy utilization for high speed indexing type applications with stability concerns set aside or placed in the background (within reason of course, while maintaining good risk management practices).

Kollmorgen designed its next-generation AKD™ series with the versatility, communications, power and bandwidth needed to build higher throughput, greater precision and more capable features into the machine.

Potential Energy Savings

Any manufacturing process, sped-up or run at a faster rate for improved throughput, requires a machine with the capability of faster response times than its previous design to maintain quality. The machine must have the capability to move and/or act on the product at a faster rate, and respond to all commands and disturbances within the limit of the product and/or process itself. From an axis perspective, the actual process-work time is often fixed and cannot be increased under an existing process technology, leaving only product or workpiece transfer times as the available time to be sped-up. This faster rate results in an increase of the specific axis' peak HP (horsepower) requirements during the acceleration and deceleration times, from its baseline production rate by the product of the increased ratio for both speed and torque.

To illustrate this concept, several high speed indexing applications were chosen for determining the inertia ratio sweet spot for the lowest power requirements (expressed herein as the '% - Energy saving versus inertia ratio).

Dynamic Applications:

The following information summarizes the results of several **high speed indexing applications** both direct drive and mechanically advantaged (belted in this case). They accomplish completely different jobs in different industries and markets, with very little friction and no external loading. In each case of the application, the process time is assumed fixed and held constant, forcing a reduced time to make a specific move (as often seen in the real world). Index times were set for a fixed peak Torque for three cases at**:** (1) ~1.6xT_rms, (2) ~2.0xT_rms and (3) ~2.4xT_rms. Once the maximum traverse RPM velocity (N) and the RMS equivalent velocity (N_rms) are calculated for each motion-profile, they are constant for that specific motion-profile − regardless of the inertia mismatch or ratio.

What one may expect for the different application cases, but not necessary intuitive, is that all the relative percentage (%) energy savings are essentially the same. That is because the theoretical maximum power savings possible for each case falls within a few percent of each other. Since the percentage (%) of the energy savings as a function of inertia ratio is relatively the same for all cases, only the second case (~2.0xT_rms) energy results is presented within this article.

As can be seen from the graph (Figure A.), a 3:1 inertia ratio (*J_load : Jm*) over a 1:1 ratio can present an actual energy savings up to approximately 39.7% (Figure B, 2^{nd} Column). Another ratio of 5:1 can present an actual energy savings upwards of approximately 47.6% (~80% of the theoretical maximum available [3^{rd} Column]), while an 8:1 ratio presents approximately 53.6% (~87.5% [4^{th} Column] of the theoretical maximum available). These examples present a pretty significant energy savings to the machine user. Now let's look at this a little deeper in relation to the *J_load : Jm* ratio.

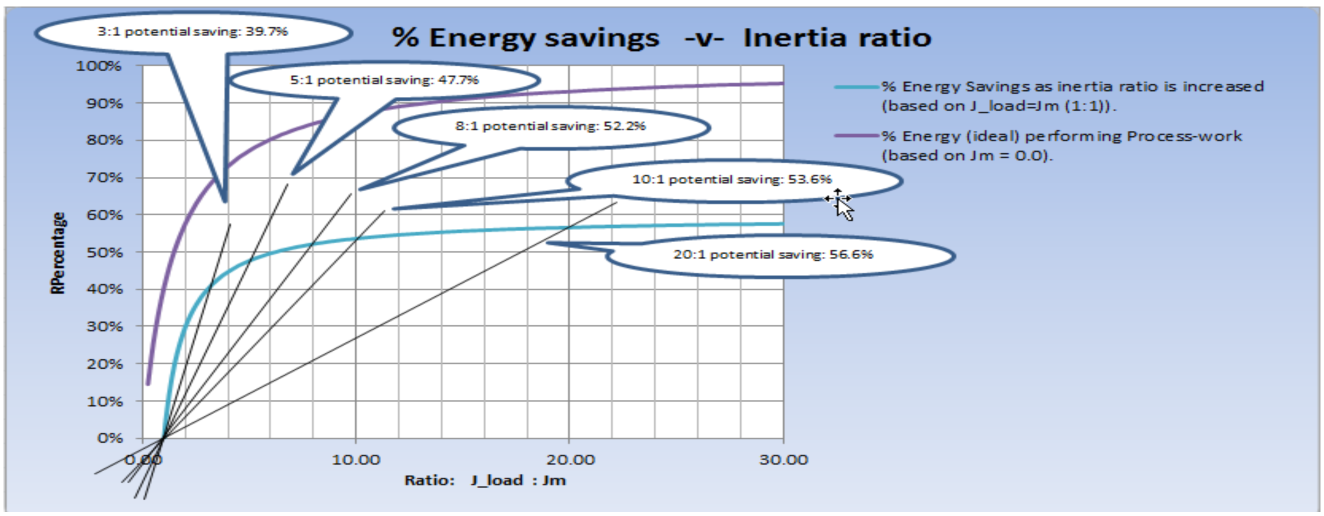

Figure A: A visual perspective of the potential energy savings from a 1:1 inertia mismatch baseline

While identifying an ideal inertia ratio or ratio range for maximum energy savings is highly subjective, a typical user wants to save as much energy as possible. So given the choice, a desirable energy saving goal would likely fall in the range of 80% to 90% (Figure B [4^{th} Column]) of the theoretical maximum available [3^{rd} Column], to upwards of 95%. The result is a desirable inertia ratio of *J_Load : Jm* = 5 to 20:1 (Figure A), a suited range for today's most dynamic applications.

Continuing upward, a range of 90-95% presents additional energy savings, translating into an inertia ratio range of 10-20:1, while an 8:1 ratio presents notable energy savings potential of 87.5% of the theoretical maximum available (Figure B & C). Many new motor-drive systems today are with the capability of accomplishing these dynamic applications with some, but little additional risk of stability issues.

Kollmorgen's AKM™ servomotor series gives unprecedented choice and flexibility, and the ability to quickly co-engineer modifications to perfectly fit any application.

It is important to remember that the energy savings for any given axis will have a direct impact on a motor's sizing / selection and cost[1], since the traverse velocity (N) and N_rms requirements are fixed by the motion profile. Hence the required RMS application Torque (T_rms) will be smaller, so a smaller less costly motor could be utilized if available.

Actual Energy Savings Comparison					
Column 1	*Column 2*		*Column 3*		*Column 4*
J_load : Jm	**Actual Energy Savings Potential**		**Theoretical Maximum Savings Potential**		**% Energy Savings Possible of Theoretical Max = $100*(1-e^{(-Ln(Load/Jm))})$**
1:1	0.00%	=	59.58%	●	0.00%
1:1.5	19.88%	=	59.58%	●	33.37%
2:1	29.79%	=	59.58%	●	50%
2.75:1	37.93%	=	59.58%	●	63.67%
3:1	39.69%	=	59.58%	●	66.62%
5:1	47.66%	=	59.58%	●	80%
8:1	52.15%	=	59.58%	●	87.53%
10:1	53.63%	=	59.58%	●	90%
15:1	55.63%	=	59.58%	●	93.37%
20:1	56.57%	=	59.58%	●	94.96%
30:1	57.57%	=	59.58%	●	96.67%
J_load/0.00	59.58%*	=	59.58%	●	100%

Figure B: Presents Actual Energy Savings Potential as a function of inertia ratio (*J_load : Jm*)

* Theoretical Maximum Energy Savings available or possible, calculated with *Jm* = 0 (*J_total* = *J_load*).
** Values within this graph came from the model where T (peak) was set = ~2.0xTrms (other models basically presented same results).

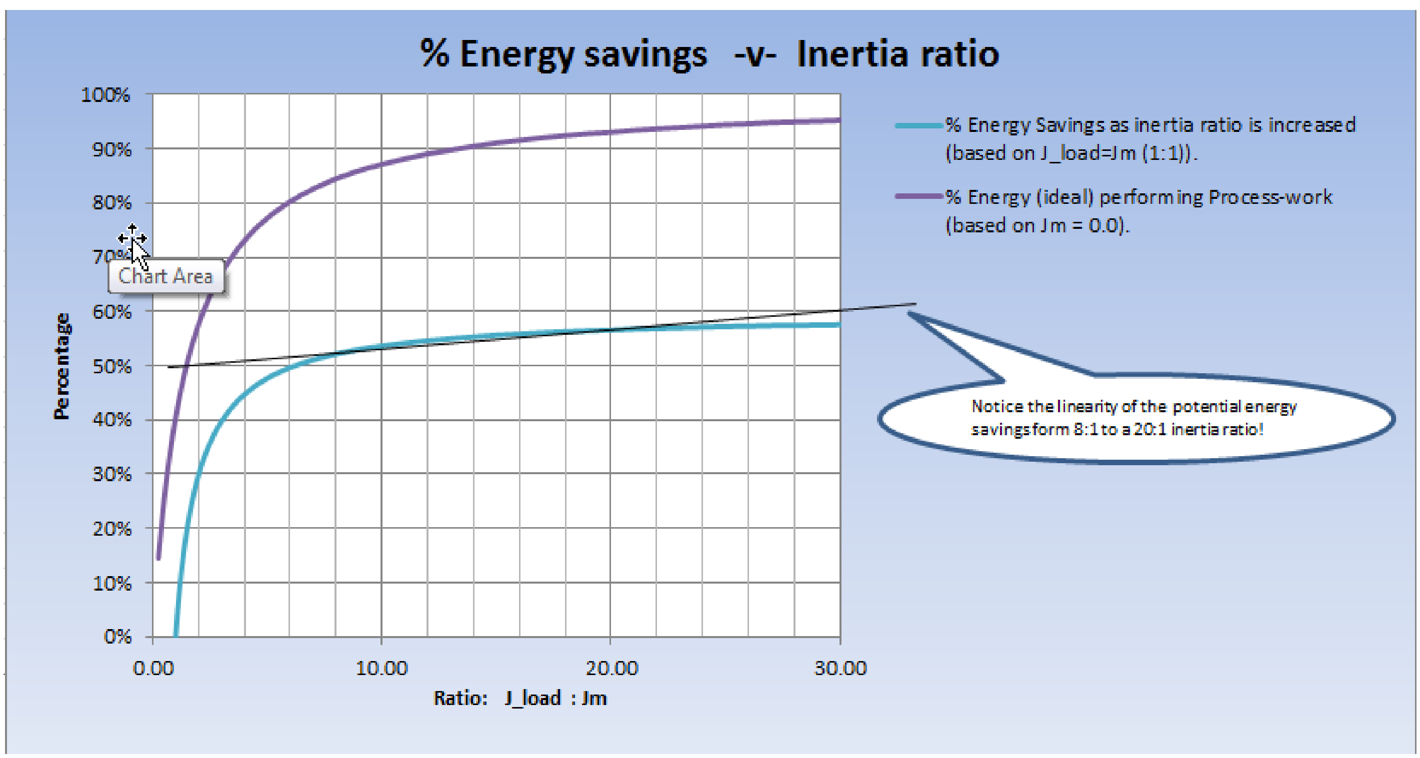

Figure C: Linearity of Potential Energy Savings as a function of the inertia ratio between 8:1 and 20:1

NOTES:

(a) The **% Actual Energy Savings Potential** for any inertia ratio can be found relative to a 1:1 inertia ratio (Figure B) with the **% Theoretical Maximum Savings Potential** calculated with *Jm* = 0 by the expression**:**

{**% Theoretical Maximum Savings Potential** (Jm = 0)}●(1-$e^{(-\ln(J\_load/Jm))}$), where the {% **Theoretical Maximum Savings Potential** (Jm = 0)} = 59.58%.

If we wanted to know the % **Actual Energy Savings Potential** with an 8:1 ratio versus a 1:1 ratio, we would solve the problem as follows**:**

% Actual Energy Savings Potential (w/8:1 ratio) = 59.58% ● (1-$e^{(-\ln(8))}$) = 59.58% ● 0.875 = 52.13%, same as determined from the actual motion profile calculations.

(b) If we had a 2:1 inertia ratio on a present axis and wanted to estimate the **% Actual Energy Savings** of going to a 15:1 inertia ratio for a high speed indexer, it could be estimated from the values of the 2nd Column (Figure B) as follows:

100●(55.63%-29.79%)/(100-29.79) = 100●25.84/70.21 = 36.8% **Energy Savings** and a 1.5 to 15**:**1 upgrade would present approximately 44.6% **Energy Savings**.

In contrast, the **% Actual Energy** increase of going to a 3:1 inertia ratio versus the present ratio of 10:1, for example, due to a product obsolescence, could be approximated from the values of 2nd Column (Figure B) as follows:

100●(39.69%-53.63)/(100-53.63) = 100●−13.94/46.37 = **−**30% savings or 30% more energy cost.

(c) If we were to have a mechanically advantaged mechanism with an initial inertia ratio design goal of 10:1. ($J_{ratio} = 10$); we can estimate the Gear Ratio (GR) to be GR = $\sqrt{(J_{load}/(J_m \bullet J_{ratio}))}$.
Thus if J_{load} = 100_Kg.cm^2 and J_m = 1.0_Kg.cm^2, then GR = $\sqrt{\frac{100}{10}} =$ 3.162, or ~3:1. Thereafter the designer should make sure the selected motor has the required speed and torque capability, or select another motor with the capability (T_rms, N_rms, T_peak, & N_max/traverse) and adjust the final GR to be in the desired J_{ratio} range. Then fine tune the numbers and motor selection for final calculations and confirmation of the application's selection.

Cost Savings Summary

Proper motor-drive feedback selection of a servo controlled axis is perhaps the single most significant savings element a machine designer can make for reducing the user's operational energy cost.

Today's digital servo drive technologies are embedded with significantly higher feedback resolution capabilities. The utilization of these capabilities with the latest high resolution feedback devices results in stable and repeatable axis control, due to their overall higher bandwidth[2] (BW) capability. This combined with a good mechatronics design, in harmony with the process and/or work to be performed by each machine axis, allows the *J_load* **:** *Jm* (factor-of-merit) to be increased considerably for most applications, compared to what was available over a decade ago.

In basic terms, machine performance and axis controllability (ease of servo control-loop tuning) will typically increase as the inertia ratio approaches 1**:**1, but lower manufacturing and operating cost (possibly lower machine cost) are achieved by the utilization of higher inertia ratios. With stability concerns less in the forefront (within reason) due to today's product advances and increased capability of the servo control components and mechanisms, inertia ratios for higher energy efficiencies can be specifically addressed. This translates to an ability to more confidently pin point an inertia ratio range in the order of ~8-20:1 for an axis' most efficient power utilization with minimal risk.

With the utilization of today's advanced servo drives paired with high resolution feedback and low inertia servomotors, most mechanically advantaged indexing applications can achieve a substantial level of energy savings. It must be noted however that many high speed indexers have a much smaller *J_load* whether mechanically advantaged or not. This is not to say that a direct drive solution cannot have a much higher inertia ratio[1], for it can in fact, by orders of magnitude: limited in many cases only to the compliance of the steel driving the load, the machine's frame stiffness, feedback resolution and available system BW[2]. These dynamic applications, whether indexing and/or providing constant high speed correction, can achieve substantial energy savings. However, the inertia load (*J_load*) of many high speed indexers can be much lower, often approaching the standard motor rotor inertia (*Jm*) for the comparable and required torque available in today's marketplace.

Kollmorgen is a pioneer in developing direct drive motor technologies that include cartridge, frameless and housed servomotors.

Our exclusive Kollmorgen Cartridge DDR® servomotors combine the performance advantages of a frameless motor with the ease of installation of a full-frame motor. The advanced electromagnetic design provides up to 50 percent more torque density than comparably sized conventional servomotors.

Detailed conclusion featured on the next page

[1] Stephens, Lee. (2010, August 12). The Significance of Load to Motor Inertia Mismatch. *www.kollmorgen.com*. Retrieved from www.kollmorgen.com/uploadedFiles/kollmorgencom/Service_and_Support/Knowledge_Center/White_Papers/KOL_MotorInertiaMismatch_Brief_08_12_10.pdf

[2] Stephens, Lee. (2007, June 21). Get on the Bandwagon with Servo Bandwidth. *Machine Design.* Retrieved from machinedesign.com/archive/get-bandwagon-servo-bandwidth

The KBM™ series offers high performance, long life and simple installation in the most compact space. Its unique design allows the motor to be directly embedded in your machine, using the machine's own bearings to support the rotor. As a result, the total number of parts count is reduced while eliminating maintenance of gearboxes, belts or pulley.

Kollmorgen Housed Direct Drive Rotary (DDR) motors offer high performance and zero maintenance in a precision servo solution. These motors combine large diameter, short length and a high number of magnetic poles to provide outstanding torque density, while eliminating the need for gearboxes, timing belts and other transmission components.

Conclusion

A figure-of-merit is generally a guideline for determining something relative to something else as an action or design guide. The inertia ratios presented within this article are to help minimize energy requirements for dynamic high speed applications, whether a direct drive design or a mechanically advantaged design.

With the advantages of today newest servo drive technologies and availability of high resolution feedback, the dance has become the balance between: (1) best possible performance with basically no consideration of operating cost and (2) best or acceptable performance designed to maximize energy management and savings. Though the mathematical solutions are well documented for the mechanical systems, they lack consideration of the actual machine function or work to be performed, while also lacking consideration of any specific control capability and/or its limitations for the best selection of a motor-drive-feedback system.

The potential user savings in power utility payments alone justifies a deeper analysis of the percentage (%) between the actual energy utilized to drive the load and the energy utilized by the motor to drive itself, for a complete evaluation of the total energy consumption. Considering the newest available technologies in harmony with the process/work to be performed, with good mechatronic axis designs, higher inertia ratios/mismatches can be chosen for lowering the machine's total energy consumption. The higher the percentage (%) of actual energy utilized to drive a load, the lower the total energy utilization will be together with lower motor torque requirements (possibly resulting in a smaller motor/drive and another cost saving).

As environmental and monetary cost of power rises, the machine designer's goal will be to select the best motor-drive-feedback system for the application while considering advantages and disadvantages of all technologies to be utilized, each considered and applied to the individual axes of the machine, for best overall machine performance. This evaluation should include the individual axis inertia ratio **(*J_load* : *Jm*)** as it applies to energy efficiencies/utilization balanced with other axis requirements. Such efforts will enhance the chance of design success by reducing the risk of failure (Risk Management) and at the same time, present a machine design that is significantly differentiated from other designs in the marketplace.

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